Uniformly accelerated mirrors. I. Mean fluxes

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The Davies-Fulling model describes the scattering of a massless field by a moving mirror in $1+1$ dimensions. When the mirror travels under uniform acceleration, one encounters severe problems which are due to the infinite blueshift effects associated with the horizons. On one hand, the Bogoliubov coefficients are ill defined and the total energy emitted diverges. On the other hand, the instantaneous mean flux vanishes. To obtain well-defined expressions we introduce an alternative model based on an action principle. The usefulness of this model is to allow us to switch on and off the interaction at asymptotically large times. By an appropriate choice of the switching function, we obtain analytical expressions for the scattering amplitudes and the fluxes emitted by the mirror. When the coupling is constant, we recover the vanishing flux. However, it is now followed by transients which inevitably become singular when the switching off is performed at late time. Our analysis reveals that the scattering amplitudes (and the Bogoliubov coefficients) should be seen as distributions and not as mere functions. Moreover, our regularized amplitudes can be put in a one to one correspondence with the transition amplitudes of an accelerated detector, thereby unifying the physics of uniformly accelerated systems. In a forthcoming article, we shall use our scattering amplitudes to analyze the quantum correlations among emitted particles which are also ill defined in the Davies-Fulling model in the presence of horizons.

INTRODUCTION

When considering the fluxes emitted by a uniformly accelerated mirror described by the Davies-Fulling model, one encounters a paradoxical situation: when working in the initial vacuum, the local flux of energy vanishes whereas the Bogoliubov coefficients encoding pair creation do not. Moreover, $\langle N_\omega \rangle$, the mean number of particles with frequency ω emitted to $\mathcal{J}^+$ diverges [1–3], and so does the corresponding total energy $\langle H \rangle = \int_0^\infty d\omega \, \omega \langle N_\omega \rangle$.

It should be pointed out that similar properties are shared by all uniformly accelerated systems. They are indeed found (in a slightly different form) in the case of a uniformly accelerated classical charge [4], an accelerated two-level atom [5–7], and for accelerated black holes [8,9]. The vanishing of the energy flux and the divergence of the total energy are both unavoidable when considering uniformly accelerated systems whose coupling to the radiation field is constant. The vanishing of the flux follows from the facts that the orbits with uniform acceleration are generated by the boost operator of the Lorentz group and that the Minkowski vacuum is a null eigenstate of this operator. Hence stationarity is built in and this implies a vanishing flux. (When applied to accelerated atoms this is known as Grove's theorem [10–13].) The divergence of the total energy emitted follows from the singular behavior of the "Rindler" modes (i.e., the eigenmodes of the boost operator) on the horizons associated with a uniformly accelerated trajectory; see Appendix C of [7] for a general proof.

In order to obtain regular expressions for the flux and the energy emitted, one needs either to regularize the trajectory by decreasing the acceleration at asymptotically large times,

or, as we shall do, to switch off the coupling between the accelerated system (here the mirror) and the radiation field. This procedure was already applied to an accelerated two-level atom in [7] and regular expressions have been found for the local flux and the total energy emitted. In the present paper, it is our intention to apply a similar treatment to an accelerated mirror. However, one immediately encounters a problem: this analysis cannot be performed within the framework of the original Davies-Fulling model because, by construction, the reflection on the mirror is total. Therefore, we first introduce an alternative model based on an action principle which, on one hand, reproduces the results of the Davies-Fulling model when the coupling between the mirror and the radiation is constant and, on the other hand, allows one to switch on and off the interactions.

When using this model to describe the scattering by an accelerated mirror, we obtain regular expressions for the transition amplitudes in the place of the singular Bogoliubov coefficients obtained with the Davies-Fulling model. Both local quantities, such as the mean flux $\langle T_{VV} \rangle$, and global ones, such as the mean energy $\langle H \rangle$ and density of particles $\langle N_\omega \rangle$, are now well defined. Moreover, as long as the coupling is constant, we recover the fact that the energy flux vanishes. However, it is now preceded and followed by transient effects associated with the switching on and off. Because of the ever increasing Doppler effect associated with constant acceleration, these effects are exponentially blueshifted (in terms of the proper time of the mirror). Therefore, in order to get a finite energy, the rate of switching off the interaction must be faster than the growth of the Doppler effect (a condition also found in [7]). If this condition is not satisfied, the mirror emits an infinite energy, thereby recovering ill-defined results like those obtained with the Davies-Fulling model.

To further clarify the physics in play in the scattering by a mirror of acceleration a, we compute the transition ampli-

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tudes governing pair creation in a "mixed" representation. By mixed we mean that in each pair one quantum is characterized by ω, a Minkowski frequency, whereas its partner is characterized by λ, a Rindler frequency, the eigenvalue of the energy with respect to the proper time of the mirror. This representation is useful for the following reasons. First, when $\lambda = \Delta M$, the scattering amplitudes closely correspond to the transition amplitudes governing the absorption and emission of photons by an accelerated two-level atom whose energy gap is ΔM. It is quite nice to see how, for each Minkowski frequency ω, the "exciton" of the atom is replaced by the emission of the partner of the Minkowski quantum. Secondly, the range of Minkowski frequencies ω that participate to the processes when the interactions last for a proper time lapse of T is finite and grows with ae^{aT}. Hence the physics in play is a succession of pair creation acts with Doppler shifted Minkowski frequencies. However, this cannot be seen by considering only the expectation value of the flux, i.e., the one-point function of the stress tensor, because these successive individual effects perfectly interfere with each order since, in the absence of recoils, the orbit is generated by a Killing field. In order to reveal these local acts, one should either consider the correlations among emitted quanta by computing the two-point function [14] of the energy flux or enlarge the dynamics so as to take into account the recoil effects. In the following article [15] we will analyze these quantum correlations, and in a forthcoming paper we shall analyze recoil effects.

I. THE DAVIES-FULLING MODEL AND ITS EXTENSIONS

In this section, we first analyze the case of asymptotically inertial trajectories (Sec. I A). We introduce notation that also applies to the cases of trajectories which enter or leave space-time through null infinities (Sec. I B), cases which turn out to be more delicate to analyze. In Sec. I C we introduce the self-interacting model. Section I D is devoted to the study of energy fluxes.

A. Asymptotically inertial trajectories

In the Davies-Fulling model [1], one studies the scattering of a massless scalar field induced by imposing a Dirichlet boundary condition along a timelike trajectory in $1+1$ dimensions. The evolution of the field operator is governed by the d'Alembert equation

$$(\partial_t^2 - \partial_z^2)\Phi(t,z) = 0, \tag{1}$$

together with the reflection condition along the trajectory of the mirror $z = z_{\rm cl}(t)$,

$$\Phi(t, z_{\rm cl}(t)) = 0. \tag{2}$$

Since we work in $1+1$ dimensions with a massless field, it is particularly useful to work in the lightlike coordinates $U, V = t \mp z$. Then, Eq. (1) becomes $\partial_U \partial_V \Phi(U,V) = 0$, and its general solution is a sum of a function of U plus a function
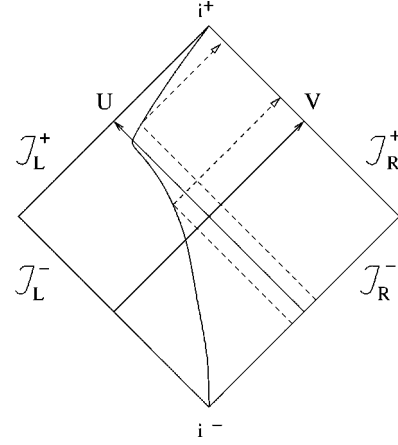

FIG. 1. In this Penrose diagram, the solid line is a timelike trajectory going from i^- to i^+. The dashed lines represent incoming V configurations which give rise to the production of a pair of outgoing U quanta (right movers).

of V. Since the trajectory of the mirror is given once for all, the recoil effects induced by the scattering of quanta are neglected.

In this subsection, in addition to the condition that the mirror trajectory be timelike, we consider only asymptotically inertial trajectories, i.e., trajectories which originate from the timelike past infinity i^- and which end in i^+ (see Fig. 1). Then, since the reflection is total, the configurations emerging from $\mathcal{J}_R^-$, the right part of $\mathcal{J}^-$, are completely decoupled from those emerging from $\mathcal{J}_L^-$. Therefore, one can analyze what happens on each side separately. On the right-hand side, all left movers are scattered into right movers and sent toward $\mathcal{J}_R^+$. In second quantization, when the trajectory is not inertial, this leads to the production of pairs formed with two right movers. Similarly, on the left-hand side, one studies the scattering from $\mathcal{J}_L^-$ to $\mathcal{J}_L^+$. Since the expressions governing the scattering on the left are obtained from those on the right by exchanging U and V, we will restrict ourselves to the analysis of the scattering from $\mathcal{J}_R^-$ to $\mathcal{J}_R^+$.

To analyze the scattering in second quantization, one first needs to identify the in basis of modes which are defined before the scattering occurs. On $\mathcal{J}_R^-$, the usual eigenmodes of Minkowski energy $i\partial_t = \omega > 0$ can be used since $\mathcal{J}_R^-$ is a Cauchy surface for the left movers when the mirror emerges from i^- (this is no longer the case for trajectories which emerge from $\mathcal{J}_R^-$; see Sec. I B). On $\mathcal{J}_R^-$ the Minkowski modes are given by

$$\varphi_\omega^{V,{\rm in}}(V, U = -\infty) = \frac{e^{-i\omega V}}{\sqrt{4\pi\omega}}, \tag{3}$$

where the upper index V,in means that the mode is left moving and defined on the initial Cauchy surface $\mathcal{J}^-$. We have introduced the index V in order to be able to deal with partially reflecting mirrors for which left and right movers

should be simultaneously considered. The norm of $\varphi_\omega^{V,\mathrm{in}}$ is determined by the usual Klein-Gordon scalar product. On $\mathcal{J}_R^-$, one has

$$(\varphi_\omega^{V,\mathrm{in}}, \varphi_{\omega'}^{V,\mathrm{in}}) \equiv \int_{-\infty}^{+\infty} dV \, \varphi_\omega^{V,\mathrm{in}*} \, i \overleftrightarrow{\partial}_V \varphi_{\omega'}^{V,\mathrm{in}} = \delta(\omega - \omega'), \quad (4)$$

where $f \overleftrightarrow{\partial}_V g = f \partial_V g - g \partial_V f$.

For finite values of U, on the right-hand side of the mirror, i.e., for $V \gtrsim V_{\mathrm{cl}}(U)$, the in mode $\varphi_\omega^{V,\mathrm{in}}$, the solution of Eq. (2) which has Eq. (3) as initial Cauchy data is

$$\varphi_\omega^{V,\mathrm{in}}(V,U) = \frac{e^{-i\omega V}}{\sqrt{4\pi\omega}} - \frac{e^{-i\omega V_{\mathrm{cl}}(U)}}{\sqrt{4\pi\omega}}. \quad (5)$$

To analyze its final frequency content, it should be Fourier decomposed on $\mathcal{J}_R^+$. In total analogy with what we have on $\mathcal{J}_R^-$, on $\mathcal{J}_R^+$ the positive frequency modes are

$$\varphi_{\omega'}^{U,\mathrm{out}}(U, V = +\infty) = \frac{e^{-i\omega' U}}{\sqrt{4\pi\omega'}}. \quad (6)$$

Since they are complete, on $\mathcal{J}_R^+$, the in modes can be written as

$$\varphi_\omega^{V,\mathrm{in}}(U, V = +\infty) = \int_0^\infty d\omega' (\alpha_{\omega'\omega}^{UV*} \varphi_{\omega'}^{U,\mathrm{out}} - \beta_{\omega'\omega}^{UV*} \varphi_{\omega'}^{U,\mathrm{out}*}). \quad (7)$$

When evaluated on $\mathcal{J}_R^+$, the overlaps are given by

$$\alpha_{\omega'\omega}^{UV*} \equiv (\varphi_{\omega'}^{U,\mathrm{out}}, \varphi_\omega^{V,\mathrm{in}}) = -2 \int_{-\infty}^{+\infty} dU \, \frac{e^{i\omega' U}}{\sqrt{4\pi/\omega'}} \frac{e^{-i\omega V_{\mathrm{cl}}(U)}}{\sqrt{4\pi\omega}}, \quad (8)$$

$$\beta_{\omega'\omega}^{UV*} \equiv (\varphi_{\omega'}^{U,\mathrm{out}*}, \varphi_\omega^{V,\mathrm{in}}) = 2 \int_{-\infty}^{+\infty} dU \, \frac{e^{-i\omega' U}}{\sqrt{4\pi/\omega'}} \frac{e^{-i\omega V_{\mathrm{cl}}(U)}}{\sqrt{4\pi\omega}}. \quad (9)$$

To interpret the scattering in terms of particle creation, one should decompose the Heisenberg field operator Φ in both the in and out bases. When working with a complex field, annihilation in operators of particles and antiparticles are defined by

$$a_\omega^{V,\mathrm{in}} = (\varphi_\omega^{V,\mathrm{in}}, \Phi), \quad b_\omega^{V,\mathrm{in}} = (\varphi_\omega^{V,\mathrm{in}}, \Phi^\dagger). \quad (10)$$

Because the in modes $\varphi_\omega^{V,\mathrm{in}}$ form an orthogonal and complete basis, these operators satisfy the canonical commutators when the field operator satisfies the equal-time commutation relation $[\Phi(t,z), \partial_t \Phi^\dagger(t,z')] = i\delta(z-z')$. The in vacuum state is defined, as usual, by $a_\omega^{V,\mathrm{in}}|0\rangle = b_\omega^{V,\mathrm{in}}|0\rangle = 0$. Similarly, the out operators are defined with the out modes $\varphi_\omega^{U,\mathrm{out}}$.

Since we are dealing with a linear theory without sources, the overlaps α and β of Eqs. (8) and (9) define the Bogoliubov coefficients relating the initial and final operators a_ω, b_ω. Therefore, these overlaps determine the expectation values (as well as the nondiagonal matrix elements) of all
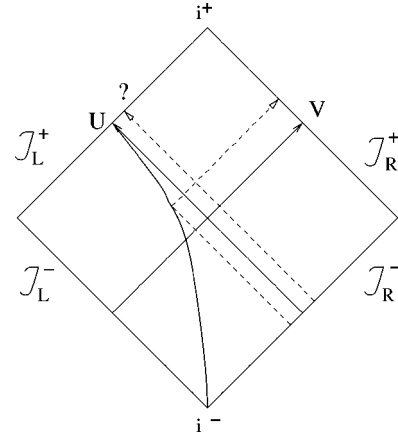

FIG. 2. In this Penrose diagram we represent the trajectory defined in Eq. (12). This trajectory has often been considered (see, e.g., [14]) because of its analogy with the Hawking effect. In this case, $\mathcal{J}_R^-$ is a Cauchy surface whereas $\mathcal{J}_R^+$ is not. The portion of $\mathcal{J}_L^+$ with $V > 0$ plays the role of the future horizon of the black hole. The dashed lines are incoming left movers. One sees that for $V < 0$ the quanta are reflected, giving rise to right movers for all values of U. On the contrary, for $V > 0$, the incoming quanta do not reach the trajectory and end up in $\mathcal{J}_L^+$. The question mark is there to bring the reader's attention to the issue of the choice of the appropriate basis of out modes to decompose the field configurations when the mirror crosses $\mathcal{J}_L^+$.

operators built with Φ. For instance, when the initial state is vacuum, the mean number of right moving particles of energy ω' received on $\mathcal{J}_R^+$ is

$$\langle N_{\omega'}^U \rangle \equiv {}_{\mathrm{in}}\langle 0 | a_{\omega'}^{U,\mathrm{out}\dagger} a_{\omega'}^{U,\mathrm{out}} | 0 \rangle_{\mathrm{in}} = \int_0^\infty d\omega |\beta_{\omega'\omega}^{UV}|^2. \quad (11)$$

B. Nonasymptotically inertial trajectories

When the trajectory does not end on i^+ (or does not begin from i^-), the strict decoupling between left and right movers is no longer realized even when the reflection on the mirror is total. Consider, for instance, the trajectory

$$V_{\mathrm{cl}}(U) = -\kappa^{-1} e^{-\kappa U}. \quad (12)$$

The mirror goes from i^- to $V = 0$ on $\mathcal{J}_L^+$. Depending on the sign of V, left movers emerging from $\mathcal{J}_R^-$ are either reflected into right movers for $V < 0$ or end as left movers on $\mathcal{J}_L^+$ for $V > 0$ (see Fig. 2). Thus, on $\mathcal{J}^+$, the image of $\varphi_\omega^{V,\mathrm{in}}$ of Eq. (3) now contains two pieces:

$$\varphi_\omega^{V,\mathrm{in}}(U,V) = \Theta(V) \frac{e^{-i\omega V}}{\sqrt{4\pi\omega}} - \frac{e^{i\omega\kappa e^{-\kappa U}}}{\sqrt{4\pi\omega}}. \quad (13)$$

This mode is singular at $V = 0$ where the mirror hits $\mathcal{J}_L^+$. On the other side of the mirror, the U in modes emerging from $\mathcal{J}_L^-$ are fully reflected into left movers but they are also singular on $\mathcal{J}_L^+$. Hence both sets of in modes are singular on $\mathcal{J}^+$, at $V = 0$.

This raises an interesting question: Given that the mirror trajectory ends on $\mathcal{J}_L^+$, which is part of the final Cauchy surface, what is the appropriate set of out modes to describe the scattered field configurations?

The procedure we shall follow to choose these modes is to decouple the radiation field asymptotically from the mirror, i.e., to make the mirror asymptotically transparent. In this case, the free Minkowski modes $e^{-i\omega'V}/\sqrt{4\pi\omega'}$ and $e^{-i\omega'U}/\sqrt{4\pi\omega'}$ still form a complete out basis. We shall not adopt the other choice, which consists in working with out modes defined on either side of the mirror on $\mathcal{J}_L^+$. These modes are so singular that their (Minkowski) energy content is not defined. Nevertheless, when working in a state specified on $\mathcal{J}^-$, the expectation values of local operators whose support is $V \neq 0$ are independent of the out basis one chooses. The out basis is necessary only for computing global quantities such as the total energy $\langle H \rangle = \int_0^\infty d\omega\, \omega \langle N_\omega \rangle$.

When adopting the asymptotic decoupling hypothesis, the image of $\varphi_\omega^{V,\text{in}}$ on $\mathcal{J}_R^+ \cup \{\mathcal{J}_L^+(V>0)\}$ can be decomposed as

$$\varphi_\omega^{V,\text{in}} = \int_0^\infty d\omega' (\alpha_{\omega'\omega}^{jV*}\varphi_{\omega'}^{j,\text{out}} - \beta_{\omega'\omega}^{jV*}\varphi_{\omega'}^{j,\text{out}*}), \quad (14)$$

where $\varphi_{\omega'}^{j,\text{out}}$ are the usual Minkowski modes, as in Eq. (7). The Bogoliubov coefficients α, β are now 2×2 matrices. The discrete index j stands for U, V and is summed over when repeated. These coefficients are still given by the Klein-Gordon scalar product as in Eqs. (8) and (9), since the out basis is composed of the usual Minkowski modes. When the trajectory is asymptotically inertial, we recover what happens in the right-hand side of the mirror for $\alpha^{ij} = \alpha^{UV}$, $\beta^{ij} = \beta^{UV}$, and on the left-hand side for $\alpha^{ij} = \alpha^{VU}$, $\beta^{ij} = \beta^{VU}$. In addition, one has $\alpha^{VV} = \beta^{VV} = \alpha^{UU} = \beta^{UU} = 0$. Finally, we mention that a similar decomposition of Eq. (14) holds on each side of the mirror when the mirror travels from $\mathcal{J}^-$ to $\mathcal{J}^+$, as is the case for a uniformly accelerated system (see Sec. II).

C. Partially transmitting mirrors

In preparation for subsequent analysis, we now present how to study partially transmitting mirrors. In this case, one should also consider U and V modes simultaneously. Indeed, when the trajectory is asymptotically inertial, an incoming U mode is partially scattered into an outgoing V mode and partially transmitted as an outgoing U mode. Hence, when the mirror is not inertial, a proper description of the Bogoliubov coefficients requires one to consider 2×2 matrices α, β which mix U and V modes.

There are two different ways to describe partially transmitting mirrors. First, one can choose from the outset the transmission coefficient (expressed in the rest frame of the mirror) and deduce from it the Bogoliubov coefficients (see Sec. II B in [16]). We shall not follow this method since it does not allow one to switch off the coupling to the radiation field.

The other method is based on self-interactions described by an action. The principal usefulness of this model is to allow switching on and off of the coupling of the radiation with the mirror. We will see in the next sections that this is necessary to obtain well-defined transition energy fluxes for a uniformly accelerated mirror. In the following, we shall use only this model (see Sec. III in [16] for more details).

In this model, the scattering on the mirror is governed by an action whose density is localized on the mirror trajectory $x_{\text{cl}}^\mu(\tau)$, where τ is the proper time,

$$L_{\text{int}} = -\int d\tau\, H_{\text{int}}(\tau) = -g_0 \int d\tau\, g(\tau) \int d^2 x$$
$$\times \delta^2[x^\mu - x_{\text{cl}}^\mu(\tau)]\mathcal{F}[\Phi^\dagger(t,z),\Phi(t,z)]. \quad (15)$$

g_0 is the coupling constant. The real function $g(\tau)$ controls the time dependence of the interaction: When the coupling lasts a proper time lapse equal to $2T$, $g(\tau)$ is normalized by $\int d\tau\, g(\tau) = 2T$. To preserve the linearity of the scattering, $\mathcal{F}$ must be a quadratic form of the field Φ, and to have a well-defined Hamiltonian, it should be Hermitian. The various possibilities with the lowest number of derivatives are $\mathcal{F}_0 = \Phi^\dagger \Phi$, $\mathcal{F}_1 = \Phi^\dagger i \overleftrightarrow{\partial}_\tau \Phi$, and $\mathcal{F}_2 = \partial_\tau \Phi^\dagger \partial_\tau \Phi$.

In the interacting picture, the charged field evolves freely. It can thus be decomposed as

$$\Phi(U,V) = \int_0^\infty \frac{d\omega}{\sqrt{4\pi\omega}}(a_\omega^U e^{-i\omega U} + a_\omega^V e^{-i\omega V} + b_\omega^{U\dagger} e^{i\omega U}$$
$$+ b_\omega^{V\dagger} e^{i\omega V}). \quad (16)$$

The annihilation and creation operators of left and right moving particles (and antiparticles) are constant and obey the usual commutation relations

$$[a_\omega^i, a_{\omega'}^{j\dagger}] = \delta^{ij}\delta(\omega - \omega'), \quad [b_\omega^i, b_{\omega'}^{j\dagger}] = \delta^{ij}\delta(\omega - \omega'). \quad (17)$$

All other commutators vanish. In the interacting picture, the states evolve through the action of a time-ordered operator $Te^{iL_{\text{int}}}$. When the initial state is vacuum, up to second order in g_0, the state on $\mathcal{J}^+$ is

$$Te^{iL_{\text{int}}}|0\rangle = |0\rangle + iL_{\text{int}}|0\rangle + \frac{(iL_{\text{int}})^2}{2}|0\rangle + |D\rangle. \quad (18)$$

The ket $|D\rangle$ contains terms arising from time ordering. None of these terms contribute to the total energy emitted (see [16] for a detailed analysis). Hence we drop $|D\rangle$ from now on.

The relationship between this model and the original Davies-Fulling model can be made explicit by considering the case where $\mathcal{F} = \Phi^\dagger i \overleftrightarrow{\partial}_\tau \Phi$ and $g(\tau) = 1$ (see [16]). In this case, whatever the mirror trajectory is, the first order transition amplitudes are related to the overlaps $\alpha_{\omega\omega'}^{ij}, \beta_{\omega\omega'}^{ij}$ entering into Eq. (14) in the following way:

$$A_{\omega\omega'}^{VV*} \equiv \langle 0|a_\omega^V e^{iL_{\text{int}}} a_{\omega'}^{V\dagger}|0\rangle_c = \delta(\omega - \omega') - ig_0\alpha_{\omega\omega'}^{VV}, \quad (19a)$$

$$B_{\omega\omega'}^{VV*} \equiv \langle 0|e^{-iL_{\text{int}}} a_\omega^{V\dagger} b_{\omega'}^{V\dagger}|0\rangle = -ig_0\beta_{\omega\omega'}^{VV}, \quad (19b)$$

$$A^{VU*}_{\omega\omega'} \equiv \langle 0|a^V_\omega e^{iL_{\text{int}}} a^{U\dagger}_{\omega'}|0\rangle_c = -ig_0 \alpha^{VU*}_{\omega\omega'}, \qquad (19c)$$

$$B^{VU*}_{\omega\omega'} \equiv \langle 0|e^{-iL_{\text{int}}} a^{V\dagger}_\omega b^{U\dagger}_{\omega'}|0\rangle = ig_0 \beta^{VU*}_{\omega\omega'}, \qquad (19d)$$

where the subscript $\langle\ \rangle_c$ means that only the connected graphs are kept. In Eqs. (19b) and (19d), one sees clearly the link between the β coefficients and pair creation amplitudes.

When using these amplitudes to compute energy fluxes one encounters severe infrared divergences due to the massless character of Φ. These divergences can be eliminated by considering the Hamiltonian with one more derivative:

$$\mathcal{F}[\Phi^\dagger, \Phi] = \frac{dx^\mu_{\text{cl}}}{d\tau} \frac{dx^\nu_{\text{cl}}}{d\tau} (\partial_\mu \Phi^\dagger \partial_\nu \Phi + \partial_\mu \Phi \partial_\nu \Phi^\dagger). \qquad (20)$$

The two terms within the parentheses mean that the interaction is symmetrical under charge conjugation. This implies that the transition amplitudes will be invariant under the exchange of a and b operators.

It should be stressed that the self-interacting model can handle without modification the cases when the trajectory enters and/or leaves space through null infinities. In these cases, when computing transition amplitudes perturbatively, one automatically adopts the convention of using asymptotically free modes. Indeed, the interacting picture is based on the assumption that the interaction is switched on and off asymptotically. This remark reinforces the well-founded character of the choice adopted in the former subsection to use, for the out basis, free Minkowski modes on $\mathcal{J}^+$.

D. Energy fluxes

In this subsection, we compute physical observables such as the number of emitted particles, the energy, and its fluxes, to second order in g_0 and when the initial state is vacuum. We study only the left moving quanta emitted toward $\mathcal{J}^+_L$; all the results for the right movers can be obtained by exchanging U and V.

The mean number of V particles of energy ω is particularly simple because only the second term of Eq. (18) contributes. One obtains

$$\langle N^V_\omega \rangle \equiv \langle 0|e^{-iL_{\text{int}}} a^{V\dagger}_\omega a^V_\omega e^{iL_{\text{int}}}|0\rangle$$

$$= \int_0^\infty d\omega' (|B^{VV}_{\omega\omega'}|^2 + |B^{VU}_{\omega\omega'}|^2), \qquad (21)$$

in the place of Eq. (11) since the partner of a^V_ω can be either a U or a V quantum. Then the (subtracted) integrated energy is, as usual,

$$\langle H^V_M \rangle = 2 \int_0^\infty d\omega\, \omega \langle N^V_\omega \rangle, \qquad (22)$$

where the factor of 2 stands for particles+antiparticles.

One can also compute the local flux of energy. The corresponding Hermitian operator is $T_{VV} = \partial_V \Phi^\dagger \partial_V \Phi + \partial_V \Phi \partial_V \Phi^\dagger$. Its expectation value is given by

$$\langle T_{VV}(V) \rangle \equiv \langle 0|e^{-iL_{\text{int}}} T_{VV} e^{iL_{\text{int}}}|0\rangle - \langle 0|T_{VV}|0\rangle$$

$$= \langle T^I_{VV} \rangle + \langle T^{II}_{VV} \rangle, \qquad (23)$$

where

$$\langle T^I_{VV} \rangle \equiv \langle 0|L_{\text{int}} T_{VV} L_{\text{int}}|0\rangle_c$$

$$= 2 \sum_{j=U,V} \int\int_0^\infty d\omega\, d\omega' \frac{\sqrt{\omega\omega'}}{2\pi} e^{-i(\omega'-\omega)V}$$

$$\times \left(\int_0^\infty dk\, B^{Vj*}_{\omega k} B^{Vj}_{\omega' k} \right) \qquad (24)$$

and

$$\langle T^{II}_{VV} \rangle \equiv -2[\text{Im}\{\langle 0|T_{VV} L_{\text{int}}|0\rangle\} + \text{Re}\{\langle 0|T_{VV} L_{\text{int}} L_{\text{int}}|0\rangle_c\}]$$

$$= -2\,\text{Re}\left\{ \sum_{j=U,V} \int\int_0^\infty d\omega\, d\omega' \frac{\sqrt{\omega\omega'}}{2\pi} e^{-i(\omega'+\omega)V} \right.$$

$$\left. \times \left(\int_0^\infty dk\, A^{Vj*}_{\omega k} B^{Vj}_{\omega' k} \right) \right\}. \qquad (25)$$

We have subtracted the average value of T_{VV} in the vacuum in order to remove the zero point energy. In Eq. (23), we introduced $\langle T^I_{VV} \rangle$, which determines the integrated (positive) energy $\langle H^V_M \rangle$ of Eq. (22) and $\langle T^{II}_{VV} \rangle$, which integrates to 0. Note that the linear term in g_0 in the first equality of Eq. (25) reappears in the second through the definition of the A terms given by Eqs. (19).

In addition to these expressions based on the amplitudes A and B, one can also express $\langle T_{VV} \rangle$ by using the "scattered" Wightman function.[1] This function is defined by

$$W(U,V;U',V') = \langle 0|e^{-iL_{\text{int}}} \Phi^\dagger(U,V) \Phi(U',V') e^{iL_{\text{int}}}|0\rangle_c. \qquad (26)$$

To obtain the subtracted flux, one needs also the unperturbed Wightman function evaluated in the vacuum:

$$W_{\text{vac}}(U,V;U',V') = \langle 0|\Phi^\dagger(U,V) \Phi(U',V')|0\rangle$$

$$= -\frac{1}{4\pi}[\ln(V'-V-i\epsilon)$$

$$+ \ln(U'-U-i\epsilon)]. \qquad (27)$$

In terms of these functions, the mean flux on $\mathcal{J}^+_L$ reads [16]

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[1]This illustrates the fact that one does not need to choose an out basis when computing expectation values of local operators with initial states prepared on $\mathcal{J}^-$. Notice, however, that the subtraction in Eq. (28) implicitly reintroduces the notion of a Minkowski vacuum on $\mathcal{J}^+$ since the *only* singularity of $W_{\text{vac}}(U,V;U',V')$ in Eq. (27) is the usual short distance one, independently of the presence of the mirror on $\mathcal{J}^+$. Thus, since subtracting the vacuum contribution on $\mathcal{J}^+$ is equivalent to subtracting that of the out modes, the use of Eq. (27) implies that the mirror is no longer coupled to the radiation field at asymptotically late times.
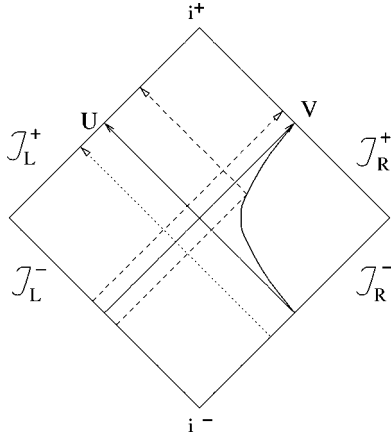
FIG. 3. In this Penrose diagram, we show a uniformly accelerated trajectory going from $V=0$ on $\mathcal{J}_R^-$ to $U=0$ on $\mathcal{J}_R^+$. The dashed lines represent the scattering of a pair of quanta (represented by localized wave packets) emerging from $\mathcal{J}_L^-$. One particle is reflected into an outgoing V quantum for $U<0$ whereas the other ends as a left mover on $\mathcal{J}_R^+$ for $U>0$. The dotted line represents a V quantum which is not reflected by the mirror.

$$\langle T_{VV}(V)\rangle = 2 \lim_{V'\to V} \partial_V \partial_{V'}[W(U,V;U',V')$$
$$- W_{\text{vac}}(U,V;U',V')]. \quad (28)$$

Notice finally that this expression also applies to the Davies-Fulling model and leads to the well-known result

$$\langle T_{VV}(V)\rangle^{\text{DF}}$$
$$= -\frac{1}{2\pi} \lim_{V'\to V} \lim_{\epsilon\to 0} \partial_V \partial_{V'}[\ln(U_{\text{cl}}(V') - U_{\text{cl}}(V) - i\epsilon)$$
$$- \ln(V' - V - i\epsilon)]$$
$$= \frac{1}{6\pi}\left\{\left(\frac{dU_{\text{cl}}}{dV}\right)^{1/2} \partial_V^2\left[\left(\frac{dU_{\text{cl}}}{dV}\right)^{-1/2}\right]\right\}. \quad (29)$$

From this equation one sees that the energy flux is local in that it contains at most three derivatives of $U_{\text{cl}}(V)$ evaluated at the advanced time V. When considering the interacting model with $g(\tau)\neq\text{const}$, this local property will be lost.

II. UNIFORMLY ACCELERATED MIRRORS

Uniform acceleration means that

$$\frac{d^2x^\mu}{d\tau^2}\frac{d^2x_\mu}{d\tau^2} = -a^2 = \text{const.} \quad (30)$$

In terms of Minkowski space-time coordinates, the trajectory reads $t^2 - z^2 = UV = -1/a^2$. In the following, we will consider a uniformly accelerated mirror living in the right Rindler wedge R, i.e., its trajectory is $U_{\text{cl}}(V) = -1/a^2V$ with V running from 0 to $+\infty$ (see Fig. 3).

The scattering associated with this trajectory leads to several difficulties when using the Davies-Fulling model. In Sec. II A, we list these difficulties. Then we will see how

they can be resolved by using our self-interacting model with a smooth switching on and off of the coupling.

A. The difficulties in using the Davies-Fulling model

When considering the scattering by a uniformly accelerated mirror with the Davies-Fulling model, on the left side of the trajectory, the images on $\mathcal{J}^+$ of the scattered in modes are

$$\varphi_\omega^{U,\text{in}}(U,V) = \Theta(U)\frac{e^{-i\omega U}}{\sqrt{4\pi\omega}} + \Theta(V)\left(-\frac{e^{i\omega/a^2V}}{\sqrt{4\pi\omega}}\right), \quad (31)$$

$$\varphi_\omega^{V,\text{in}}(U,V) = \Theta(-V)\frac{e^{-i\omega V}}{\sqrt{4\pi\omega}}. \quad (32)$$

As expected from Sec. I B, they are singular where the mirror enters ($V=0$) and leaves ($U=0$) the space-time. Their overlaps with plane waves (out modes) are [2]

$$\alpha_{\omega'\omega}^{VV*} \equiv (\varphi_{\omega'}^{V,\text{out}}, \varphi_\omega^{V,\text{in}}) = \frac{1}{4\pi}\frac{\omega+\omega'}{\sqrt{\omega\omega'}}\frac{i}{\omega-\omega'}, \quad (33a)$$

$$\beta_{\omega'\omega}^{VV*} \equiv (\varphi_{\omega'}^{V,\text{out}*}, \varphi_\omega^{V,\text{in}}) = \frac{1}{4\pi}\frac{\omega-\omega'}{\sqrt{\omega\omega'}}\frac{i}{\omega+\omega'}, \quad (33b)$$

$$\alpha_{\omega'\omega}^{VU*} \equiv (\varphi_{\omega'}^{V,\text{out}}, \varphi_\omega^{U,\text{in}}) = \frac{-1}{\pi a}K_1\left(-i\frac{2\sqrt{\omega\omega'}}{a}\right), \quad (33c)$$

$$\beta_{\omega'\omega}^{VU*} \equiv (\varphi_{\omega'}^{V,\text{out}*}, \varphi_\omega^{U,\text{in}}) = \frac{-i}{\pi a}K_1\left(\frac{2\sqrt{\omega\omega'}}{a}\right), \quad (33d)$$

where $K_1(z)$ is a modified Bessel function (see Appendix A).

When computing $\langle N_\omega\rangle$ [given by $\int_0^\infty d\omega' (|\beta_{\omega'\omega}^{VV}|^2 + |\beta_{\omega'\omega}^{VU}|^2)$; see Eqs. (11) and (21)], the integral over ω' diverges in the infrared. Moreover, $\alpha_{\omega'\omega}^{VV}$ diverges when $\omega = \omega'$. Similarly, $\alpha_{\omega'\omega}^{VU}$ is ill defined since the integral representation of the Bessel function in Eq. (33c) requires one to contain a finite and positive real part [see Eq. (A1)].

In addition to these problems in momentum space, when computing the space-time properties of the flux, one encounters the following properties. When plugging $U_{\text{cl}}(V) = -1/a^2V$ for $V>0$ into Eq. (29), one finds that $\langle T_{VV}(V)\rangle^{\text{DF}}$ vanishes.[2] This is not in agreement with the nonvanishing character of the β since, on one hand, $\langle H_M^V\rangle$

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[2]Generally, the trajectories which provide a vanishing V flux are of the form $U_{\text{cl}}(V) = (AV+B)/(CV+D)$ [1]. Timelikeness imposes $AD>BC$. If $C=0$, we recover inertial trajectories. If $C\neq 0$, we recover uniformly accelerated trajectories with $a = C/\sqrt{AD-BC}$.

$= \int_0^\infty d\omega \, \omega \langle N_\omega^V \rangle$ and, on the other hand, $\langle H_M^V \rangle = \int_{-\infty}^{+\infty} dV \langle T_{VV} \rangle$. It is as if the created particles were carrying no energy flux [1,17,18].

To sum up, the difficulties of the Davies-Fulling model for uniformly accelerated trajectories are as follows.

(i) Unregulated and infrared diverging overlaps, Eqs. (33).

(ii) A diverging expression for $\langle N_\omega \rangle$, the mean number of particles created.

(iii) A vanishing local flux, although pair creation transition amplitudes do not vanish.

(iv) When one considers the scattering by *two* uniformly accelerated mirrors with symmetrical trajectories, i.e., which both obey $UV = -1/a^2$, the (unregulated) overlaps β also vanish, together with the local flux. From [19], one could infer that these settings form a perfect interferometer. This cannot be exactly the case since the two mirrors live in two causally uncorrelated regions. This issue will be fully discussed in a forthcoming paper [15].

B. The switching function g

To avoid the difficulties listed above, we shall use our self-interacting model, based on Eqs. (15) and (20). We require that the switching function $g(\tau)$ be continuous, differentiable, and it decrease sufficiently rapidly for large τ. A choice we find very convenient and shall adopt is

$$g(\tau) = e^{-2\eta \cosh(a\tau)} = e^{-\eta[aV_{cl}(\tau) + 1/aV_{cl}(\tau)]}, \quad (34)$$

where $0 < \eta \ll 1$ is a dimensionless parameter which controls the switching time.

$g(\tau)$ can be interpreted in two different ways: either as governing how the interactions with the mirror are turned on and off, or as a mathematical regulator which properly defines the transition amplitudes $A_{ij}^{\omega\omega'}, B_{ij}^{\omega\omega'}$. This second interpretation implicitly relies on the limit $\eta \to 0$ (see Appendix B). In the body of the text, we shall use the first (physical) interpretation of $g(\tau)$.

The features of $g(\tau)$ are the following (see Fig. 4).

(a) A plateau of height 1 centered around $\tau = 0$ and of width equal to

$$2T \equiv 2\pi g_{\lambda=0} \simeq \frac{2|\ln(2\eta)|}{a}. \quad (35)$$

(b) Slopes which are maximal and equal to $a/e + \mathcal{O}(\eta^2)$ for $a\tau \simeq \pm(aT + \ln 2)$.

(c) An exponentially decreasing tail. We shall see that this extremely rapid decreasing behavior ($\sim e^{-\eta e^{a|\tau|}}$) is sufficient for having a finite Minkowski flux on $\mathcal{J}^+$.

We now compute the Rindler and Minkowski Fourier components of g since they will reappear in the expressions of the transition amplitudes. They are given in terms of the modified Bessel functions:

$$g_\lambda \equiv \frac{1}{2\pi} \int_{-\infty}^{+\infty} d\tau \, e^{-2\eta \cosh(a\tau)} e^{i\lambda\tau} = \frac{1}{a\pi} K_{i\lambda/a}(2\eta), \quad (36)$$
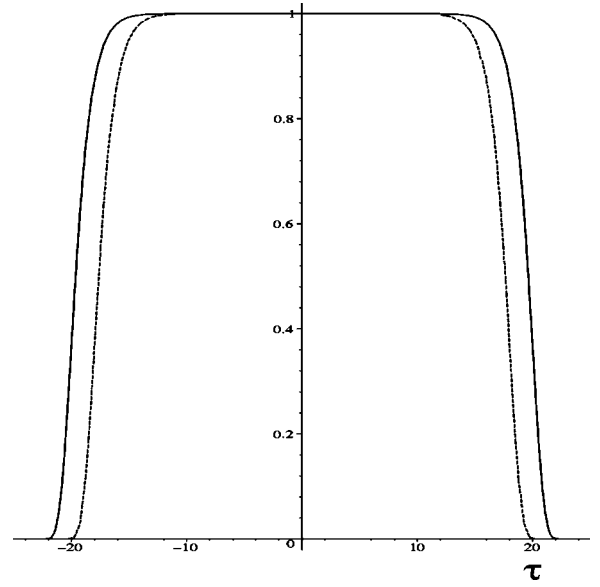

FIG. 4. Here are presented two $g(\tau)$, both for $a = 1$. The solid line is with $\ln \eta = -20$ and the dashed line with $\ln \eta = -18$. One sees that the size of the plateau is linear in $\ln \eta$ whereas the slope is independent of η. This remark will be crucial when considering the Rindler energy emitted by the mirror (see Sec. III).

$$g_\omega \equiv \frac{1}{2\pi} \int_0^\infty dV \, e^{-\eta(aV + 1/aV)} e^{i\omega V}$$

$$= -\frac{1}{a\pi} \frac{1}{\sqrt{1 - i\omega/a\eta}} K_1(2\eta\sqrt{1 - i\omega/a\eta}). \quad (37)$$

In the ultraviolet, for $\lambda \gg a$ and $\omega \gg a/\eta$, the Rindler and Minkowski components decrease, respectively, as

$$|g_\lambda| \overset{\lambda\to\infty}{\sim} \frac{1}{a}(a/\lambda)^{1/2} e^{-\pi\lambda/2a}, \quad (38)$$

$$|g_\omega| \overset{\omega\to\infty}{\sim} \frac{1}{a}(a/\omega)^{3/4} \eta^{1/4} e^{-\sqrt{2\omega\eta/a}}. \quad (39)$$

From Eq. (38), one sees that the ultraviolet behavior of the Rindler component is independent of η. This is a direct consequence of point (b), which states that the maximal slope of the switching function is independent of η when expressed in proper time τ. From Eq. (39) one finds instead that the ultraviolet behavior of the Minkowski components is damped by the regulator η.

C. Regularized amplitudes and particle content

Given $g(\tau)$ of Eq. (34), the transition amplitudes can be explicitly calculated. They are given in Appendix B and, as expected, they are well defined. Nevertheless, we will not work with these amplitudes characterized by Minkowski frequencies since they are not convenient for computing the expectation value of observables. Similarly, we will not work with the transition amplitudes with Rindler frequencies even though they are simply expressed in terms of g_λ of Eq. (36).

It turns out that it is more convenient to express the fluxes and the energy in terms of transition amplitudes containing one Minkowski and one Rindler frequency. More precisely, these amplitudes mix Minkowski and "Unruh" quanta. The Unruh modes $\hat{\varphi}_\lambda^j$ are linear combinations of positive frequency Minkowski modes *and* eigenmodes of Rindler energy λ (see [5,13] and Appendix C).

These "mixed" transition amplitudes[3] are given by the matrix elements of the scattering operator with the Minkowski operator a_ω^j and the Unruh one $\hat{a}_\lambda^j$:

$$A_{\omega\lambda}^{ij*} \equiv \langle 0|a_\omega^i e^{iL_{\text{int}}}\hat{a}_\lambda^{j\dagger}|0\rangle_c,$$

$$B_{\omega\lambda}^{ij} \equiv \langle 0|a_\omega^i \hat{b}_\lambda^j e^{iL_{\text{int}}}|0\rangle. \tag{40}$$

To first order in g_0, using Eqs. (15), (20), (63), and (78), we get

$$A_{\omega\lambda}^{VU*} = -\frac{ig_0}{\pi a}\sqrt{\omega\lambda/(1-e^{-2\pi\lambda/a})}(1-i\omega/a\eta)^{-(1-i\lambda/a)/2}$$

$$\times K_{1-i\lambda/a}(2\eta\sqrt{1-i\omega/a\eta}), \tag{41}$$

$$B_{\omega\lambda}^{VU} = \frac{ig_0}{\pi a}\sqrt{\omega\lambda/(1-e^{-2\pi\lambda/a})}(1-i\omega/a\eta)^{-(1+i\lambda/a)/2}$$

$$\times K_{1+i\lambda/a}(2\eta\sqrt{1-i\omega/a\eta}). \tag{42}$$

Moreover, as shown in Appendix C, the U and V Unruh modes coincide when evaluated along the trajectory. Therefore $B_{\omega\lambda}^{VU} = B_{\omega\lambda}^{VV}$, and similarly for A.

Using these amplitudes, the mean Minkowski energy emitted to $\mathcal{J}_L^+$ can be written

$$\langle H_M^V\rangle = 4\int_0^\infty d\omega\,\omega\int_{-\infty}^{+\infty}d\lambda|B_{\omega\lambda}^{VV}|^2$$

$$= \int_0^\infty d\omega\int_{-\infty}^{+\infty}d\lambda\,h_M(\omega,\lambda;\eta). \tag{43}$$

The number 4 before the first integral means that processes involving U or V particles and U or V antiparticles equally contribute to the mean Minkowski energy emitted to $\mathcal{J}_L^+$ (or $\mathcal{J}_R^+$).

The exact computation of $\langle H_M^V\rangle$ will be performed in the next section. In the following, we shall study $h_M(\omega,\lambda,\eta) = 4\omega|B_{\omega\lambda}^{VV}|^2$ to give a qualitative understanding of $\langle H_M^V\rangle$. The usefulness of the mixed representation is that the behavior of $h_M(\omega,\lambda;\eta)$ in the (ω,λ,η) space is quite easy to explain. The starting point is that the norm of $B_{\omega\lambda}^{VU}$ decreases as $e^{-2\pi|\lambda|/a}$ for $|\lambda|\gg a$, as expected from the correspondence mentioned in footnote 3. Hence, the relevant range of λ is

centered around 0 and of extension a few a's. When λ belongs to this interval, the following analysis applies.

First, η acts as a regulator: the Minkowski frequencies which contribute to h_M belong to the interval

$$a\eta \lesssim \omega \lesssim \xi a/\eta, \tag{44}$$

where ξ is a numerical factor. Its value is ~ 0.50 when one uses the mid height criterion: $h_M(\omega=\xi a/\eta,\lambda;\eta)/h_M(\omega=a,\lambda;\eta)=1/2$ with λ belonging to the relevant interval.

Secondly, within the range given in Eq. (44), h_M hardly depends on ω, as shown in Fig. 5.[4] Hence, for any given value of η, one can first trivially perform the integral over ω from $a\eta$ to $\xi a/\eta$. The value of this integral is given by $h_M(\omega=a,\lambda;\eta)\times\xi a/\eta$, since $\eta\ll 1$.

Thirdly, the height of the plateau hardly depends on η (see Fig. 6). This can be understood from Eq. (A4): when Eq. (44) is satisfied and when $\eta\ll 1$, $B_{\omega\lambda}^{VU}$ is independent of η. Hence, one can take the limit $\eta\to 0$ to estimate how h_M depends on λ. In this limit, the scattering amplitudes obey

$$B_{\omega\lambda}^{VV} \stackrel{n\to 0}{\to} -ig_0\frac{\lambda}{e^{\pi\lambda/a}-e^{-\pi\lambda/a}}\gamma_{\lambda\omega}^{V*}, \tag{45}$$

where $\gamma_{\lambda\omega}^V = (\varphi_\omega^V,\hat{\varphi}_\lambda^V)$ is given in Eq. (C6) of Appendix C. In this case, we get

$$h_M(\omega=a,\lambda;\eta) = 4\omega|B_{\omega\lambda}^{VV}|^2$$

$$= \frac{g_0^2}{2\pi a}\left(\frac{\lambda}{e^{\pi\lambda/a}-e^{-\pi\lambda/a}}\right)^2, \tag{46}$$

thereby recovering the above mentioned Boltzmann expansion law for $|\lambda|\gg a$. Thus the mean energy can be approximated by

$$\langle H_M^V\rangle = \xi\frac{a}{\eta}\int_{-\infty}^{+\infty}d\lambda\,h_M(\omega=a,\lambda;\eta)$$

$$= \xi\frac{g_0^2 a^3}{6\pi\eta}. \tag{47}$$

Although the numerical factor is not exactly determined (because of the ambiguity of defining ξ), we will see in the next section that the factor $g_0^2 a^3$ and the scaling in $1/\eta$ correctly define the behavior of $\langle H_M^V\rangle$ when $\eta\ll 1$ (the exact value of ξ is 3/8 instead of 1/2).

So far, by the introduction of the switching off function [Eq. (34)], we have solved the first two problems listed in Sec. II A. Indeed, the transition amplitudes (expressed in the

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[3]These scattering amplitudes make contact with the transition amplitudes of a uniformly accelerated detector coupled to the radiation field Φ [5,6,7,13,20]. For instance, the spontaneous emission amplitude of a two-level atom with an energy gap ΔM is equal to $(\omega\sqrt{\lambda})^{-1}B_{\omega,\lambda}^{VU}|_{\lambda=\Delta M}$; see Eq. (2.48) in [13].

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[4]In this, we recover what was found for an accelerated detector; see Sec. 2.4 of [13]. In that case, the Minkowski frequencies that contribute to the processes are the Doppler frequencies that resonate with the energy gap ΔM when the interactions are turned on, i.e., those that satisfy $\omega=e^{a\tau}\Delta M$ for $-T<\tau<T$. Moreover, within that range, the transition amplitudes do not depend on ω and the limit $\eta\to 0$ can be used to estimate the amplitude, as in Eq. (45).
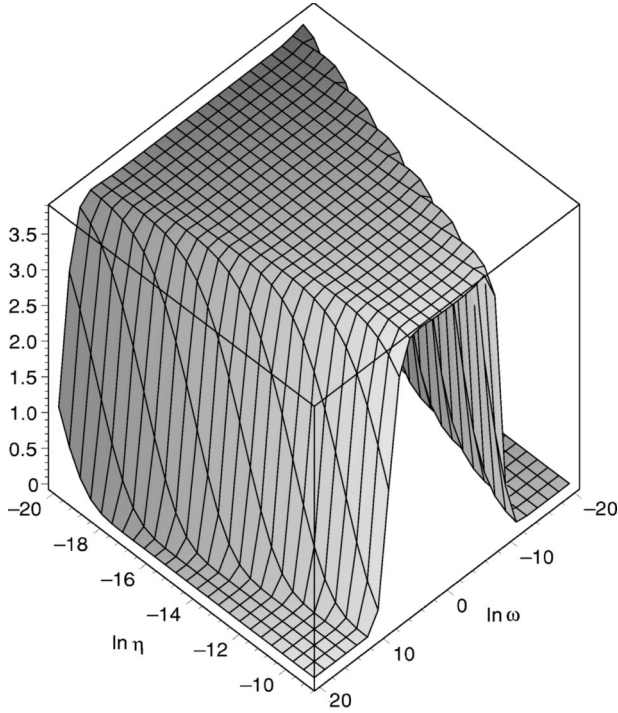
FIG. 5. Here we show $h_M(\omega,\lambda=0.15;\eta)$ in terms of $\ln \omega$ and $\ln \eta$. One sees clearly that the surface exhibits a "plateau" of constant height which is limited by the lines $\ln \omega = \pm \ln \eta$. $h_M(\omega,\lambda;\eta)$ is given in arbitrary units and $a=1$.

Minkowski, Rindler, or mixed representation) are now well defined in both the infrared and the ultraviolet, and the energy emitted is no longer infinite. The third and last point, i.e., the issue concerning the local fluxes, is the subject of the next section.

III. LOCAL MINKOWSKI AND RINDLER FLUXES

In the Davies-Fulling model, the flux is given by Eq. (29). It is a local function of the trajectory $U_{cl}(V)$ and its derivatives expressed at the advanced time V. This result relies on the time independence of the coupling. Indeed, this feature no longer occurs when the coupling to the radiation field is switched on and off.

To compute $\langle T_{VV} \rangle$ of Eq. (23) we first put together the two terms quadratic in g_0. This is appropriate when computing local properties in space-time because it leads to a simplification since this gives rise to a commutator which is local. Then the flux reads

$$\langle T_{VV}(V) \rangle = -2 \, \text{Im}\{\langle 0|T_{VV}L_{\text{int}}|0\rangle\}$$

$$+ \text{Re}(\langle 0|L_{\text{int}}[T_{VV},L_{\text{int}}]|0\rangle) \qquad (48)$$

$$= \langle T_{VV}(V) \rangle_1 + \langle T_{VV}(V) \rangle_2. \qquad (49)$$

(Note that all disconnected diagrams automatically cancel in this expression.) Both terms are governed by the second derivative of the Wightman function, Eq. (27),
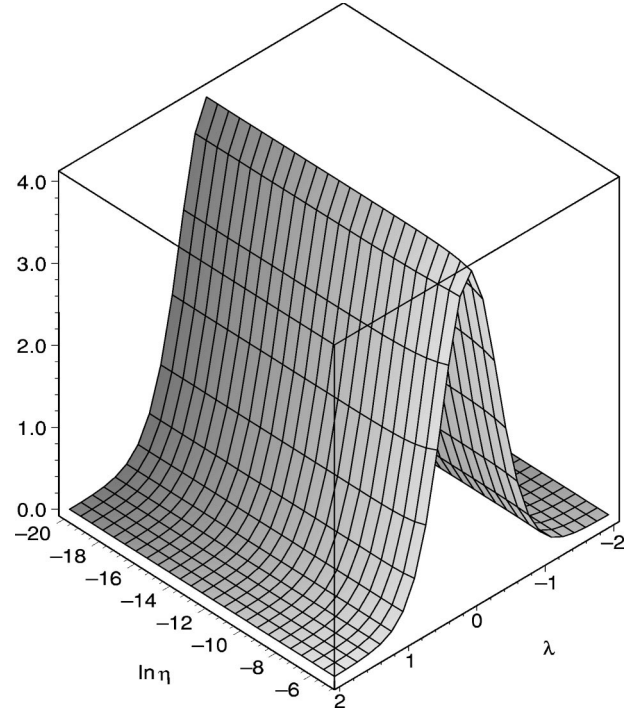

FIG. 6. Here we show $h_M(\omega=1,\lambda;\eta)$ in terms of λ and $\ln \eta$. One clearly sees that the energy density h_M is independent of η. This comes from that the fact that we compute it "within the plateau," i.e., for $a\eta \ll \omega = a \ll a/\eta$. Again, $h_M(\omega,\lambda;\eta)$ is given in arbitrary units and $a=1$.

$$\partial_V \partial_{V'} W_{\text{vac}}(V-V') = -\frac{1}{4\pi} \frac{1}{(V-V'-i\epsilon)^2}. \qquad (50)$$

Using this function, they read

$$\langle T_{VV}(V) \rangle_1 = -\frac{g_0}{2\pi^2}$$

$$\times \text{Im}\left\{ \int_{-\infty}^{+\infty} d\tau \, g(\tau) \dot{V}_{cl}^2(\tau) \frac{1}{[V-V_{cl}(\tau)-i\epsilon]^4} \right\}, \qquad (51)$$

$$\langle T_{VV}(V) \rangle_2 = \frac{g_0^2}{2\pi^2} \, \text{Re}\left\{ \int_{-\infty}^{+\infty} d\tau' \frac{g(\tau')}{[V_{cl}(\tau')-V-i\epsilon]^2} \right.$$

$$\times \int_{-\infty}^{+\infty} d\tau \, g(\tau) \partial_V \delta(V_{cl}(\tau)-V)$$

$$\times \left(\frac{i}{[V_{cl}(\tau')-V_{cl}(\tau)-i\epsilon]^2} \right.$$

$$\left. \left. + \frac{\dot{U}_{cl}(\tau)\dot{U}_{cl}(\tau')}{\dot{V}_{cl}(\tau)\dot{V}_{cl}(\tau')} \frac{i}{[U_{cl}(\tau')-U_{cl}(\tau)-i\epsilon]^2} \right) \right\}, \qquad (52)$$

where an overdot designates a derivation with respect to the proper time. The function $\partial_V \delta$ comes from the commutator

$$[\partial_V \Phi^\dagger, \partial_{V'} \Phi] = [\partial_V \Phi, \partial_{V'} \Phi^\dagger] = \frac{i}{2} \partial_V \delta(V - V'). \quad (53)$$

To evaluate the integrals, it is appropriate to use the dummy variable $\widetilde{V} = V_{\mathrm{cl}}(\tau)$ and to define a new function

$$G(\widetilde{V}) = \dot{V}_{\mathrm{cl}}(\tau[\widetilde{V}]) g(\tau[\widetilde{V}]), \quad (54)$$

which can be interpreted as the effective coupling constant when using $\widetilde{V}$ as the time. Using this function, one can evaluate Eqs. (51) and (52) by integrating by parts until the exponent of the pole is unity. All boundary contributions vanish if $g(\tau)$ decreases faster than $e^{-a|\tau|}$, a condition satisfied by the switching function we chose in Eq. (34). If $g(\tau)$ decreases more slowly than $e^{-a|\tau|}$, the expectation value of T_{VV} is ill defined. Hence it appears that the condition $g(\tau) e^{a|\tau|} \to 0$ for $\tau \to \pm\infty$ is a necessary condition for having well-defined Minkowski expressions.

Concerning Eq. (51), after three integrations by parts, the last integration is trivially performed by using $\mathrm{Im}\{(x - i\epsilon)^{-1}\} = \pi\delta(x)$. Concerning Eq. (52), the two terms within the parentheses are equal. In order to compute this expression, one first performs the integral over τ by using the function $\partial_V \delta$. Then, as for Eq. (51), one integrates by parts until one gets a first order pole.

Grouping the results for Eqs. (51) and (52), one obtains, for $V > 0$,

$$\langle T_{VV}(V>0) \rangle = \frac{1}{12\pi} [g_0 \partial_V^3 G - g_0^2 (G \partial_V^4 G + 2 \partial_V G \partial_V^3 G)] \quad (55)$$

$$= \frac{g_0^2}{12\pi} \{ (\partial_V^2 G)^2 + \partial_V [\cdots] \}. \quad (56)$$

For $V < 0$, one gets $\langle T_{VV} \rangle \equiv 0$ as expected since the $V < 0$ part of $\mathcal{J}_L^+$ is causally disconnected from the mirror trajectory.[5]

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[5] We would like to briefly comment on causality. When computing the flux in a causally disconnected point with respect to the trajectory, one must obviously find zero [20]. This is trivially the case when one expresses, in Eqs. (24) and (25), the transition amplitudes as integrals over the proper time τ and first performs the integral over ω. However, in the absence of a regulator, one loses causality when inverting the order of the integrations. This can be seen from the unregulated amplitudes where the fact that the mirror was in the R or L quadrant is lost [see Eqs. (B4)] in Appendix B. The advantage of $g(\tau)$ is to give rise to transition amplitudes [see Eqs. (B1)], wherein the prescription of the pole governed by η keeps control of causality. The same remark applies to the amplitudes in the mixed representation defined in Eqs. (41) and (42). When using them to compute the flux, causality is kept. Because of the analogy with the transition amplitudes of an accelerated detector, causality is also preserved by regularizing these amplitudes [see Eqs. (25), (26), and (63) of [7]].

In Eq. (56), we have separated the flux into two parts, a square term which will lead to a positive Minkowski energy $\langle H_M^V \rangle = \int_{-\infty}^{+\infty} dV \langle T_{VV} \rangle$, and a total derivative which does not contribute to it [note the similarity with $\langle T_{VV}^I \rangle$ and $\langle T_{VV}^{II} \rangle$ of Eq. (23)]. When using the coupling function of Eq. (34) and Eqs. (56) and (A1), one can obtain an analytical expression for $\langle H_M^V \rangle$:

$$\langle H_M^V \rangle = \frac{g_0^2 a^3}{6\pi} [8\eta^3 K_0(4\eta) + (4\eta^2 + 2\eta^4) K_1(4\eta)$$
$$- 8\eta^3 K_2(4\eta) - 3\eta^4 K_3(4\eta) + \eta^4 K_5(4\eta)]. \quad (57)$$

When taking the limit $\eta \to 0$ one obtains

$$\langle H_M^V \rangle \overset{\eta \to 0}{\sim} \frac{g_0^2 a^3}{16\pi} \frac{1}{\eta} + \mathcal{O}(1). \quad (58)$$

Hence, up to a numerical factor, we recover the result of Eq. (47). It is interesting to see how the pathological features of constant coupling reemerge when taking $\eta \to 0$. In this limit, the effective coupling constant of Eq. (54) obeys $G(V) = aV$. Hence Eq. (55) gives a vanishing flux whereas $\langle H_M^V \rangle$ clearly diverges [see Eq. (58)].

To complete the analysis of the transients, we now compute the Rindler flux $[\langle T_{vv} \rangle \equiv (dV/dv)^2 \langle T_{VV} \rangle]$ in terms of the Rindler advanced time $v = (1/a)\ln(aV)$. This analysis clearly establish that the Rindler energy carried by the transient effects is insensitive to the duration of the interaction and depends only on the rate of switching on and off the interactions. From Eq. (55), we get

$$\langle T_{vv}(v) \rangle = \frac{a^2}{12\pi} \{ -g_0 \partial_v g + g_0^2 [g \partial_v^2 g + 2(\partial_v g)^2] \}$$
$$- \frac{1}{12\pi} [-g_0 \partial_v^3 g + g_0^2 (g \partial_v^4 g + 2 \partial_v g \partial_v^3 g)] \quad (59)$$

$$= \frac{g_0^2}{12\pi} [a^2 (\partial_v g)^2 + (\partial_v^2 g)^2] + \partial_v [\cdots]. \quad (60)$$

As for the Minkowski energy $\langle H_M^V \rangle$, Eq. (60) shows that one also obtains a positive Rindler energy $\langle H_R^V \rangle = \int_{-\infty}^{+\infty} dv \langle T_{vv} \rangle$. Using g given by Eq. (34), the Rindler energy is

$$\langle H_R^V \rangle = \frac{g_0^2 a^3}{2\pi} \eta^2 K_2(4\eta), \quad (61)$$

$$\langle H_R^V \rangle \xrightarrow{\eta \to 0} \frac{g_0^2 a^3}{16\pi} + \mathcal{O}(\eta). \quad (62)$$

In Fig. 7, we have plotted the Rindler flux when the switching function g is given by Eq. (34). We previously noticed that the slope of $g(\tau)$ does not depend on η. As the slope determines the Fourier content of g_λ, one understands

why we obtain a nonvanishing Rindler energy even in the limit $\eta \rightarrow 0$. We could have chosen different switching functions $g(\tau)$ such that their slope would tend to zero. In the limit, we would have found $\langle H_R \rangle = 0$, as for a constant coupling. However, in this case, we would necessarily have obtained a diverging Minkowski energy $\langle H_M^V \rangle$, since the condition $g(\tau) e^{a|\tau|} \rightarrow 0$ for $\tau \rightarrow \pm \infty$ would not have been satisfied. Hence $\langle H_R \rangle$ cannot be sent to 0 if one requires a finite $\langle H_M^V \rangle$.

CONCLUSIONS

By considering the self-interacting model defined by Eq. (15), with $\mathcal{F}$ given in Eq. (20) and $g(\tau)$ given in Eq. (34), we have solved all the difficulties listed in Sec. II A: The transmission amplitudes are well defined and given in Eqs. (41), (42), and (B1), the mean energy flux is given in Eq. (57), and the local flux in Eq. (55). All these quantities are regularized by the parameter η which controls the switching on and off of the coupling through Eq. (34).

The important lesson which emerges from this analysis is the following: When expressing $\langle T_{VV} \rangle$ in terms of A and B as in Eqs. (24) and (25), the regulator η should be sent to 0 *after* having integrated over k, ω, and ω'. In this, we recover what was found in [22] when evaluating the energy density in the Rindler vacuum. If instead one first sends $\eta \rightarrow 0$, the unregulated expressions of the scattering amplitudes are so poorly defined that one even loses causality and crossing symmetry (see Appendix B). Therefore, in the presence of horizons, or more generally when the mirror enters or leaves space-time through null infinities, it is mandatory to consider the scattering amplitudes as distributions and not only as functions of frequencies belonging to $[0, +\infty]$.

In addition, by expressing the scattering amplitudes in the mixed representation [see Eqs. (41) and (42)], we have made contact with the physics of a uniformly accelerated detector. Indeed, its absorption or emission transition amplitudes are given by the same functions as the scattering amplitudes in the mixed representation. This strict correspondence establishes that the physics of uniformly accelerated systems is dominated by the kinematics, namely, (near) stationarity with respect to proper (Rindler) time and (near) singular behavior due to the exponentially growing blueshift effects associated with uniform acceleration.

APPENDIX A: BESSEL FUNCTIONS

In this appendix we recall some features of the modified Bessel functions $K_\nu(z)$, where $(\nu, z) \in \mathcal{C}$ (see [21], p. 374). They can be expressed by the following integral representation:

$$K_\nu(z) = \int_0^\infty dt \, e^{-z \cosh(t)} \cosh(\nu t), \qquad \text{(A1)}$$

where $|\arg(z)| < \pi/2$. For $k \in \mathcal{N}$ and $\nu \in \mathcal{R}$, one has

$$\left(\frac{1}{z} \frac{\partial}{\partial z} \right)^k \{ z^\nu K_\nu(z) \} = e^{-i\pi k} z^{\nu-k} K_{\nu-k}(z), \qquad \text{(A2)}$$
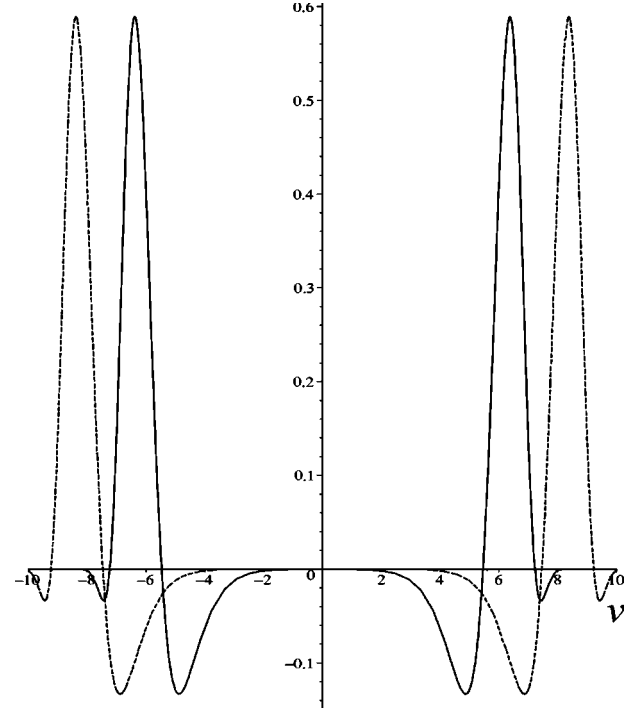

FIG. 7. Here is plotted $\langle T_{vv}(v) \rangle$ as a function of $av = \ln(aV)$, for $a = 1$ in arbitrary units and for two different values of η. The switching function has been taken for $\ln \eta = -6$ (solid curve) and $\ln \eta = -8$ (dashed curve). Notice the two following properties. The flux is significantly nonzero within the transients only. The amplitude of these transients is independent of the duration of the interactions governed by η.

$$\left(\frac{1}{z} \frac{\partial}{\partial z} \right)^k \{ z^{-\nu} K_\nu(z) \} = e^{i\pi k} z^{-\nu-k} K_{\nu+k}(z). \qquad \text{(A3)}$$

We also recall the asymptotic behavior of the K's for small and large arguments:

$$K_0(z) \overset{z \rightarrow 0}{\sim} -\ln(z)$$

and

$$K_\nu(z) \overset{z \rightarrow 0}{\sim} \frac{\Gamma(\nu)}{2} \left(\frac{2}{z} \right)^\nu \quad \text{for } \mathfrak{R}(\nu) > 0 \qquad \text{(A4)}$$

whereas

$$K_\nu(z) \overset{z \rightarrow +\infty}{\sim} \sqrt{\pi/2z} \, e^{-z} \quad \text{for all } \nu. \qquad \text{(A5)}$$

APPENDIX B: REGULARIZED TRANSITION AMPLITUDES

The aim of this appendix is to give the exact expressions for the regularized transition amplitudes in terms of Minkowski frequencies. The main virtue of the regulator η is to define them without ambiguity. The direct evaluation of Eqs. (19) with $g(\tau)$ defined by Eq. (34) gives

$$A^{VV*}_{\omega\omega'} = \delta(\omega-\omega') - \frac{4ig_0}{\pi} \frac{\sqrt{|\omega\omega'|}}{a} \frac{\eta^2}{X^2} K_2(X) \quad \text{where}$$

$$X = 2\eta\sqrt{1-i(\omega-\omega')/a\eta}, \tag{B1a}$$

$$B^{VV}_{\omega\omega'} = \frac{4ig_0}{\pi} \frac{\sqrt{|\omega\omega'|}}{a} \frac{\eta^2}{X'^2} K_2(X') \quad \text{where}$$

$$X' = 2\eta\sqrt{1-i(\omega+\omega')/a\eta}, \tag{B1b}$$

$$A^{VU*}_{\omega\omega'} = -\frac{ig_0}{\pi} \frac{\sqrt{|\omega\omega'|}}{a} K_0(Y) \quad \text{where}$$

$$Y = 2\sqrt{(\omega/a+i\eta)(-\omega'/a-i\eta)}, \tag{B1c}$$

$$B^{VU}_{\omega\omega'} = \frac{ig_0}{\pi} \frac{\sqrt{|\omega\omega'|}}{a} K_0(Y') \quad \text{where}$$

$$Y' = 2\sqrt{(\omega/a+i\eta)(\omega'/a-i\eta)}. \tag{B1d}$$

Two important remarks should be made. Because these amplitudes have been regularized, they possess analytical properties which guarantee that they obey crossing symmetry, that is,

$$A^{ij*}_{\omega\omega'} = -B^{ij}_{\omega,\omega'e^{i\pi}} \tag{B2}$$

(see [23] for exploiting this symmetry in studying accelerated detectors). Secondly, had the mirror followed the accelerated trajectory in the left quadrant rather than in the right one, the corresponding transition amplitudes would have been obtained by simply replacing η by $-\eta$.

We wish also to stress that $g(\tau)$ can be considered as a mathematical regulator which properly defines the transition amplitudes. Consider, for instance, $B^{VU}_{\omega\omega'}$. Using Eq. (34), it is given by

$$B^{VU}_{\omega\omega'} \propto \frac{\sqrt{\omega\omega'}}{a} \int_0^\infty dV \frac{1}{V} e^{i[(\omega+i\eta)V-(\omega'-i\eta)/a^2V]}. \tag{B3}$$

One clearly sees that the integral is now well defined for both $V \to 0$ and $V \to \infty$.

Finally, it is interesting to take the limit $\eta \to 0$ to see how one recovers the singular amplitudes that one would have obtained with a constant coupling. Using Eqs. (B1), in the limit $\eta \to 0$, we get

$$A^{VV*}_{\omega\omega'} \to \delta(\omega-\omega') + \frac{ig_0}{2\pi} a \frac{\sqrt{|\omega\omega'|}}{(\omega-\omega')^2}, \tag{B4a}$$

$$B^{VV}_{\omega\omega'} \to -\frac{ig_0}{2\pi} a \frac{\sqrt{|\omega\omega'|}}{(\omega+\omega')^2}, \tag{B4b}$$

$$A^{VU*}_{\omega\omega'} \to -\frac{ig_0}{\pi} \frac{\sqrt{|\omega\omega'|}}{a} K_0\left(-2i\frac{\sqrt{|\omega\omega'|}}{a}\right), \tag{B4c}$$

$$B^{VU}_{\omega\omega'} \to \frac{ig_0}{\pi} \frac{\sqrt{|\omega\omega'|}}{a} K_0\left(2\frac{\sqrt{|\omega\omega'|}}{a}\right). \tag{B4d}$$

Although the choice of our Lagrangian, based on Eq. (20), has removed the infrared divergences, the A terms are clearly ill defined, as in the original Davies-Fulling model. More importantly, crossing symmetry and causality are both lost if one uses these unregulated amplitudes. This clearly establishes that the Bogoliubov coefficients must be conceived as *distributions*, or at least as analytical functions of ω and ω', and not merely as functions defined from $[0, +\infty[$. Thus the limit $\eta \to 0$ should be performed only at the end of the calculation, after having performed all integrations. This is because the limit $\eta \to 0$ in general does not commute with these integrations.

APPENDIX C: THE UNRUH MODES

By definition, the "Unruh" [5] modes $\hat{\varphi}^V_\lambda$ and $\hat{\varphi}^U_\lambda$ possess the following properties: they are made of positive Minkowski frequency modes only, whatever is the sign of λ, and they are eigenfunctions of $iaV\partial_V$ (or $-iaU\partial_U$) with eigenvalue λ. They are thus well adapted to study uniformly accelerated systems since they are eigenmodes of $i\partial_\tau = \lambda$ where τ is the proper time calculated along the accelerated trajectory.

Since the Unruh modes form a complete and orthonormal basis, one can define in a canonical way the corresponding annihilation and creation operators of particles and antiparticles $\hat{a}^i_\lambda$, $\hat{a}^{i\dagger}_\lambda$, $\hat{b}^i_\lambda$, and $\hat{b}^{i\dagger}_\lambda$:

$$\hat{a}^i_\lambda = (\hat{\varphi}^i_\lambda, \Phi), \quad \hat{b}^i_\lambda = (\hat{\varphi}^i_\lambda, \Phi^\dagger), \tag{C1}$$

where the subscript i stands as before for U and V. Hence, the scalar field can be decomposed as

$$\Phi(U,V) = \sum_{i=U,V} \int_{-\infty}^{+\infty} d\lambda(\hat{\varphi}^i_\lambda \hat{a}^i_\lambda + \hat{\varphi}^{i*}_\lambda \hat{b}^{i\dagger}_\lambda). \tag{C2}$$

Note that the integrals over λ cover the entire real axis.

The Unruh modes are analytically expressed as follows:

$$\hat{\varphi}^V_\lambda(V) \equiv \lim_{\epsilon\to 0} \frac{[a(V-i\epsilon)]^{-i\lambda/a}}{\sqrt{4\pi\lambda(1-e^{-2\pi\lambda/a})}} \tag{C3}$$

$$= \int_0^\infty d\omega\, \gamma^V_{\lambda\omega} \frac{e^{-i\omega V}}{\sqrt{4\pi\omega}}, \tag{C4}$$

with

$$\gamma^V_{\lambda\omega} \equiv (\varphi^V_\omega, \hat{\varphi}^V_\lambda) \tag{C5}$$

$$= \frac{\Gamma(-i\lambda/a)}{\sqrt{a\pi/\lambda}\,\sinh(\pi\lambda/a)} \left(\frac{\omega}{a}\right)^{i\lambda/a} \frac{e^{-\omega\epsilon}}{\sqrt{2\pi a\omega}}$$

$$= [(\gamma^V)^{-1}_{\lambda\omega}]^*. \tag{C6}$$

Notice that the regulator ϵ in Eqs. (C3) and (C6) plays a role similar to η in the text: ϵ is *needed* to properly define the energy density in the Rindler vacuum [22].

When considering U modes, we get similar expressions with $\gamma_{\lambda\omega}^U = \gamma_{\lambda\omega}^{V*}$. Finally, when evaluated along the accelerated trajectories, within the right (R) or the left (L) quadrant, U and V Unruh modes coincide and are given by, in R,

$$V = V_{\text{cl}}^R(\tau) = e^{a\tau}/a,$$

$$U = U_{\text{cl}}^R(\tau) = -e^{-a\tau}/a,$$

and $\quad \hat{\varphi}_\lambda^V(V) = \hat{\varphi}_\lambda^U(U) = \dfrac{e^{-i\lambda\tau}}{\sqrt{4\pi\lambda(1-e^{-2\pi\lambda/a})}}, \quad$ (C7)

and in L,

$$V = V_{\text{cl}}^L(\tau) = -e^{-a\tau}/a,$$

$$U = U_{\text{cl}}^L(\tau) = e^{a\tau}/a,$$

and $\quad \hat{\varphi}_\lambda^V(V) = \hat{\varphi}_\lambda^U(U) = \dfrac{e^{i\lambda\tau}}{\sqrt{4\pi\lambda(e^{2\pi\lambda/a}-1)}}.$ (C8)

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